A Brief Survey of Factoring Techniques

To ***factor*** an expression or number means to write it as a ***product*** of several terms. This means that you are "un-multiplying"; remember that the word "factor" is also used to describe a term in multiplication. For example, we know that 3 x 5 = 15.

Here, 3 and 5 are called *factors*, and 15 is called the *product*. 3 x 5 is called a *factorization* of 15.

Can you find three other factorizations of 15?

Just from understanding what factoring means, we get a crucial fact:

You can always check to see if you have the correct answer by multiplying.

This will be very important for us while we learn and practice techniques of factoring.

We have done factoring with numbers before, using things like *factor trees*.

This sheet will give some guidelines for factoring ***variable expressions***.

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1. ***Try to find a (greatest) common factor for all terms first.***

Look for a number that divides evenly into each term, whether it is positive or negative. Also, a good rule of thumb is to use the ***least exponent*** of a variable as a common factor.

Example $-3x^2y^3 + 9xy^4 - 12x^3y^2$

Solution

3 (or –3) will divide evenly into each number, while x and y^2 are the lowest variable parts.

Therefore our greatest common factor (GCF) can be $3xy^2$ *or* $-3xy^2$

To factor out the GCF, we can divide each term by it and leave what's left inside parentheses, like this:

$$\frac{-3x^2y^3}{3xy^2} + \frac{9xy^4}{3xy^2} - \frac{12x^3y^2}{3xy^2} \quad \textit{or} \quad \frac{-3x^2y^3}{-3xy^2} + \frac{9xy^4}{-3xy^2} - \frac{12x^3y^2}{-3xy^2}$$

$$3xy^2(-xy + 3y^2 - 4x^2) \quad \textit{or} \quad -3xy^2(xy - 3y^2 + 4x^2)$$

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2. ***Try to apply a formula.***

Here are some oft-used formulae:

$A^2 + 2AB + B^2 = (A + B)^2$
$A^2 - B^2 = (A + B)(A - B)$
$A^2 - 2AB + B^2 = (A - B)^2$

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$
$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

The hard thing about using a formula is to "fit" an expression to it.

Example $y^3x^6 - 8w^9$

Solution

One guess would be that we could apply $A^3 - B^3$, since we see y^3 in the first term, subtraction, and 8, which is 2^3, in the second term.

Also, all of the powers are multiples of 3, so that settles it. Now we have to determine what to use as A and B in order to apply the formula.

We can say that $A = yx^2$, since we can check that $A^3 = y^3x^6$. Also, we can say that $B = 2w^3$, since we can check that $B^3 = 8w^9$. Now, we "plug in" A and B to the right side of the formula:

$$y^3x^6 - 8w^9 = (yx^2)^3 - (2w^3)^3 = (yx^2 - 2w^3)[(yx^2)^2 + (yx^2)(2w^3) + (2w^3)^2]$$
$$\boldsymbol{A^3 - B^3 = (A - B)(A^2 + AB + B^2)}$$

We finish by simplifying:

$$y^3x^6 - 8w^9 = (yx^2 - 2w^3)(y^2x^4 + 2yx^2w^3 + 4w^6)$$

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3. ***If no formula applies:*** this usually means there are three or more terms. If there are only two terms, a formula will (almost) always apply.

a. ***Four (or any even number more than 4) terms:*** try to use the ***Grouping Method:***

Example $5x^3 + x^2 + 15x + 3$

Solution

We want to "group" together the most similar pairs of terms, and factor those pairs. Here, I might group like this: $5x^3 + 15x \;+\; x^2 + 3$ *I did this because of the 5 and 15.*

Now, I check to see if each pair has a common factor. The first pair does: $5x$. The second pair only has a common factor of 1. Now I extract the common factors:

$5x(\boldsymbol{x^2 + 3}) + 1(\boldsymbol{x^2 + 3})$ *Check my work.*

Since the $(x^2 + 3)$ "matches" for each pair, I can factor it out as well!

When I do that, here's what is left:

$(\boldsymbol{x^2 + 3})(5x + 1)$ *Do you see why?*

This is a tricky method to use, and it often requires some "trial & error" before you get the correct factorization. The key is this: **after you factor the GCF from each pair, what's left inside should match.** If it doesn't, check your work, or try a different grouping.

b. ***Three terms (Trinomial):*** Try to use the ***"Reverse-FOIL" Method:***

Example $x^2 - 5x + 4$

<u>Solution</u>

First, write it as a product: $(x \quad)(x \quad)$ *since* $(x)(x) = x^2$

Now, we need to find numbers to fill in the "blanks." What numbers?

Remember from multiplication (FOIL) that the numbers **must have a product of 4.** So we can start by listing <u>factors</u> of 4:

$4 = (4)(1), (2)(2), (-4)(-1), (-2)(-2)$ *Do I really need the positive ones? Why?*

The numbers we need are –4 and –1, since they <u>add</u> to give the coefficient of x, -5*. We can test by using the FOIL method: $(x - 4)(x - 1)$

F $(x)(x) = x^2$
O $(x)(-1) = -x$
I $(-4)(x) = -4x$
\+ L $(-4)(-1) = 4$

$x^2 - 5x + 4$

*--This part **does not** work unless it's <u>just</u> x^2. Why not, you ask?

Let's try one like that as a comparison.

Example $2x^2 + 7x + 6$

<u>*Solution*</u>

Here we use a clever application of the grouping method.

First, take the product of the leading coefficient, 2, and the constant term, 6. 2(6)= 12, so now we look for **two factors of 12** that **add to give 7**...this is again reversing the FOIL method, as you might guess. The two factors we need are 4 and 3, and now we rewrite the expression with those factors involved:

$2x^2 + 7x + 6 = 2x^2 + 4x + 3x + 6$ *4x + 3x = 7x... and now group it!*

$2x(x + 2) + 3(x + 2) = (2x + 3)(x + 2)$ *Check my work.*

The difference I referred to above is that here, the two numbers that eventually "show up" in the factorization are 3 and 2, which do **not** add to give the coefficient of *x*, 7.

Notice, though that the (eventually grouped) addends of 7, which were 3 and 4, came from 12, which was the first step: multiplying the leading coefficient and the constant.

The standard form of a quadratic trinomial is $Ax^2 + Bx + C$, and that means step one was finding *AC*. That gives this multi-step technique its name: the ***"AC" Method.***

My point is that you need to prepare to use both techniques to have a chance at factoring trinomials successfully.

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4. ***Check your work by multiplying.***

Tedious, but effective...do it when you have time!

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5. ***If none of this applied to what you are trying to factor:***

Take a mathematics course at Napa Valley College. ☺